30-1 Field Near A Straight Wire

**Direction of the field.** The field created by a current carrying wire is circular and perpendicular to the direction of current flow.

![Diagram of magnetic field lines]

**Right Hand Rule** - Thumb in the direction of current flow, finger tips point in the direction of field at any point around the wire.

![Diagram of right hand rule]

**Magnitude of the Magnetic Field Near a Long Straight Wire**

Last year we gave you the formula for the magnetic field created near a long straight wire:

\[
B = \frac{\mu_0 i}{2\pi r}
\]
Now that your math is more sophisticated, I can show you where it comes from.

Suppose we wish to determine the field strength at point P, a perpendicular distance R from the wire. As with electric fields, we will divide the wire into small differential segments, dx, determine the field each creates at point P and sum these via integration.

Each segment dx has a current i, so we define a i dx as a \textit{current-length segment}. This segment is located a distance r from point P, and we wish to calculate the field dB it produces. This is found using the formula:

\[
\frac{dB}{4\pi} = \frac{\mu_o \ i dx \sin \theta}{r^2}
\]

where \(\mu_o\) is the permeability of free space and \(\theta\) is the angle between the direction of current flow in dx and the direction of r to point P.

But we can also write this as:

\[
\frac{dB}{4\pi} = \frac{\mu_o \ i dx \sin \theta r}{r^3}
\]

and \(dx \sin \theta r\) is \(dx \times r\) giving:

\[
\frac{dB}{4\pi} = \frac{\mu_o \ i dx x \ r}{r^3}
\]

This is known as the \textbf{Law of Biot-Savart}.

Now, because the two sides are symmetrical, and all the fields will point into the page at point P, we can integrate from 0 to \(\infty\) and double the result.

\[
B = 2 \int_0^\infty dB = \frac{\mu_o i}{4\pi} \int_0^\infty \frac{\sin \theta dx}{r^2}
\]

But \(x, \theta\) and \(r\) are not independent, so we must get \(\theta\) and \(r\) in terms of \(x\) and \(R\)!

\(r^2 = x^2 + R^2\) and \(\sin \theta = R/(x^2 + R^2)\) therefore:

\[
B = \frac{\mu_o i}{2\pi} \int_0^\infty \frac{R dx}{(x^2 + R^2)^{3/2}}
\]

Following through on the integration:

\[
B = \frac{\mu_o i}{2\pi R} \left( \frac{x}{(x^2 + R^2)^{1/2}} \right)_0^\infty = \frac{\mu_o i}{2\pi R}
\]
Example 30-1) Determine the magnitude and direction of the magnetic field 20 cm to the left of the current carrying wire shown to the right.

**Magnetic Field Due To a Circular Arc of Wire.**

The diagram to the right shows a portion of a circular loop of wire. Point P is now a uniform distance R from the wire, in the center of curvature of the loop. This actually makes the integration simpler. All we need do is find the field created by our current-length segment, let's call that i ds this time, and integrate over the entire angle of the arc (φ)!

\[ B = \frac{\mu_0 i}{4\pi R} \int_{\phi} d\phi = \frac{\mu_0 i \phi}{4\pi R} \]

Remember, angles must be in radians!

Example 30-2) The diagram to the right shows a segment of wire that consists of a circular arc of radius 20 cm and central angle of 90° and two straight sections. The wire carries a current of 12.5 A.

Determine the magnitude and direction of the magnetic field at the center of the arc.

What is the formula for the field intensity at the center of a complete loop?
30-2 Parallel Current Carrying Wires

When two current carrying wires are placed next to each other, each sits in the others magnetic field. This means that there will be a force created on each wire.

Suppose the wires carry current in the same direction as shown to the right. The right wire is in a magnetic field pointing downward created by the left wire. Using the open hand rule we see that the force created on the wire is to the left! The left wire is in a field pointed upwards created by the right wire, so the force on it will be to the right!

RULE - When two parallel wires carry current in the same direction they are attracted to each other. If the currents are antiparallel they will be repelled!

Magnitude - The magnitude of the force between the wires can be found using the force on a wire formula from chapter 29. \( F = iL \times B = iLB \sin \theta \). But now the magnetic field is created by the other wire \( B = \frac{\mu_0 i}{2\pi R} \). Plugging this in for \( B \) we get:

\[
F = \frac{\mu_0 i_1 i_2 L}{2\pi d}
\]

where the i’s are the currents in each wire, L is the length that they run parallel and d is the distance between them.

Example 30-3) The diagram to the right shows two parallel wires with current running in opposite directions. \( i_a = 3.0 \text{ A} \) and \( i_b = 4.2 \text{ A} \). The wires are separated by a distance of 10 cm.

A) Are the wires attracted or repelled from each other?

B) What is the magnitude of the force per meter created between the wires?

C) What is the magnitude and direction of the net magnetic field midway between the wires?

D) In terms of \( i_a \), \( i_b \), d and standard constants, set up an equation to determine the magnetic field at any point x from the top wire.
30-3 Amperes Law

Just as we used Gauss’s Law to determine the net electric field near several charges we can use Ampere’s Law to find the net magnetic field in situations where some symmetry exists!

Gauss’s Law

\[ \oint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \]

Here we integrated over the surface area enclosing the charge.

Ampere’s Law

\[ \oint B \cdot ds = \mu_0 i_{\text{enc}} \]

Here we integrate of the loop enclosing the current.

Suppose we wish to use ampere’s Law to determine the net magnetic field through the Ampere-Loop set up to the right. We only need to consider the current flows enclosed in the loop. Next we divide the loop into differential loop segments that are tangential to the loop at all points. Each of these will make some angle \( \theta \) to the actual magnetic field at that point. We do not actually know the direction of \( B \) at any point, but we do know that it is in the plane of the page.

Now using Amperes Law:

\[ \oint B \cdot ds = \oint B \cos \theta ds = \mu_0 i_{\text{enc}} \]

To evaluate the \( i_{\text{enc}} \) we use our right hand one more time. Placing your fingers in the direction of the integration Ampere loop, if the current in the wire points in the direction of your thumb it is positive, if the opposite direction it is negative. So in this case \( i_{\text{enc}} = (i_2 - i_1) \). That means that the equation for the net filed in this problem becomes:

\[ \oint B \cos \theta ds = \mu_0 (i_2 - i_1) \]
Revisiting a Long Straight Wire

Using Ampere’s Law to determine field near the wire.

Suppose we want to find the magnetic field strength a distance $r$ from a wire carrying a current into the page as shown. We can set up a circular Amperian loop with a radius $r$ around the wire.

We know that the field should have the same strength at all points $r$ from the wire so $B$ is a constant. Now we integrate around the loop in a counterclockwise direction (chosen arbitrarily).

Using the right hand rule we see that $B$ is parallel to $ds$ so $\theta = 0$ and $\cos\theta = 1$. Plugging back into Ampere’s Law we get:

$$\oint B \cdot ds = \oint B \cos \theta ds = \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

so

$$B = \frac{\mu_0 i}{2\pi r}$$

i is positive (hand rule) so:
Field Inside the Wire

There is a magnetic field created inside the wire as well. If we assume a uniform current density, then the current enclosed by our Amperian loop will be \( i \pi r^2/\pi R^2 \). If we plug this in for \( i \) we get:

\[
\oint B \cdot ds = \mu_0 i_{enc} = \frac{\mu_0 \pi r^2}{\pi R^2}
\]

\[
B(2\pi r) = \frac{\mu_0 \pi r^2}{\pi R^2}
\]

\[
B = \left( \frac{\mu_0 i r}{2\pi R^2} \right)
\]

Example 30-4) If the wire above has a diameter of 5 mm and carries a current of 16 A, what is the magnitude of the magnetic field 1 mm from the center?

What is the field intensity at the center of the wire?

30-4 The Solenoid

Although the field created by a single loop of wire is relatively weak, strong electromagnets can be produced by wrapping the wire many times around a hollow or solid core. This is called a solenoid. The fields created by each loop of wire will be cumulative in the center of the loops!
North End of the Solenoid Rule - Grasp the solenoid with your fingers going around the solenoid in the direction of the current flow. Your thumb will point to the north end.

Magnitude of the Field - \( B = \mu_0 in \) \( n = \) number of ampere turns per meter \( (n = N/L) \)

Example 30-5)
A solenoid is produced by wrapping 400 turns of wire around a hollow tube 12 cm long. 4.5 A of current are then passed through the wires. (Copy diagram from the board)

Label the north and south ends.

Determine the strength of the solenoid.

Field Near a Loop of Wire

We know that the field at the center of the loop is found using: \( B = \mu_0 i/2R \), but what if we are some distance \( z \) away from the center of the coil along the central axis?

The equation for this situation becomes:

\[
B_{(z)} = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}
\]
If more than one loop is present (tightly wrapped) the field intensity created by one loop is multiplied by the number of loops (N) giving:

\[ B_{(z)} = \frac{N \mu_0 i R^2}{2(R^2 + z^2)^{3/2}} \]

Example 30-6) A coil of wire is made up of 200 loops each 2.5 cm in radius. What is the strength of the magnetic field 10 cm to the right of the loop if 3.0 A of current flow through the wire?